HEAT CALCULATION FOR THE COVER OF AN ELECTROLYTIC BATH

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A heat calculation based on the assumption of isothermality sometimes leads to large errors. Local overheating can affect the normal operation of a component. Hence, an accurate heat calculation requires the solution of the heat-conduction equation, which is difficult for components of complicated shape. This paper gives an example of such a calculation.

In some electrolytic baths the steel cover is a current-conducting part and serves to distribute the current to the anodes. In this case busbars are fitted to the steel cover and connect adjacent electrolytic baths in series. The cover is a steel plate with steel ribs welded onto it lengthwise. The cover has rows of holes which are arranged symmetrically relative to the ribs and serve for the passage of the anodes through the cover. The ribs have contacts which transmit the current from the cover to the anodes. The underside of the cover and the regions around the holes on the top surface are rubber-coated. The outer busbar is connected to the cover between points A and B (Fig. 1). The current delivered to an anode depends on the distance of the particular anode from the cathode. This distance is adjusted from the reading of current-measuring probes. In the ideal case the currents delivered to the anodes should be equal. If the current is not distributed uniformly to the anodes equalizing currents are set up in the cover. The heat released in the cover is dissipated by convection and radiation by the flat surfaces and ribs and is removed by conduction through the cover to adjacent, less heated regions, from which it escapes to the surroundings. This process leads to a nonuniform temperature distribution on the cover. The temperature should be highest in region I (Fig. 1). Since part of the cover is rubber-coated, it is essential to keep the coating intact by creating conditions for the release and removal of heat from the cover so that the temperature of the hottest parts does not exceed the limit for stability of the rubber coating. The heat calculation for a cover with all these factors taken into account presents great technical difficulties.

Digital electronic computers can be used to calculate the temperature profile along the cover by solving the heat-conduction equation with due regard to the variable heat transfer.

An accurate heat calculation for the cover of an electrolytic bath necessitates the solution of the heat-conduction equation for a body of complex shape (Fig. 1) with distributed heat sources and sinks. However, for an approximate upper estimate of the possible temperatures in the case of a symmetrical current distribution the problem can be reduced to a one-dimensional one. In fact, the maximum temperature should occur in the central cross section of the cover.

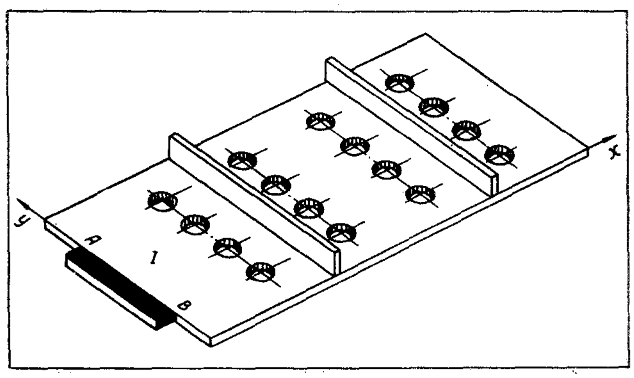

Fig. 1. Diagram of cover of R-100 electrolytic bath.

Hence, in this section the derivative of the temperature along the cover will be 0 (necessary condition for a maximum). Hence, if we cut out the central cross section of the cover (Fig. 1) we can assume that the temperature in this section is independent of the y coordinate and the heat flow through the end faces (in the direction along the bath) is zero, i. e., we can assume that the end faces are thermally insulated.

Thus, the problem is reduced to the determination of the temperature profile in a bar of variable cross section with lateral projections (ribs) (Fig. 2). This problem requires the solution of a system of three one-dimensional heat-conduction equations (we are interested only in the steady-state solution and, hence, we put $\partial T_i/\partial t = 0$)

$$\frac{d}{dx_i}\left[S(x_i)\lambda(T_i)\frac{dT_i}{dx_i}\right] - k(T_i)S_1(x_i)(T_i - T_0) + \frac{I_i^2\rho(T_i)}{S(x_i)}\eta = 0, \qquad i = 1, 2, 3 \tag{1}$$

with the corresponding boundary conditions

$$\lambda\frac{dT_i}{dx_i}\bigg|_{x_i=a_i} + f_i(T_i, x_i)\bigg|_{x_i=a_i} = 0,$$

$$a_1 = 0, \quad a_2 = l_2, \quad a_3 = l_3, \quad i = 1, 2, 3. \tag{2}$$

The equations of the system are connected with one another by the condition that the total heat flux at branch points A and B (Fig. 2) is zero. Mathematically this condition is written as follows:

$$\lambda \frac{dT_1}{dx_1}\bigg|_{x_1=a-0} - \lambda \frac{dT_1}{dx_1}\bigg|_{x_1=a+0} - \lambda \frac{dT_2}{dx_2}\bigg|_{x_2=0} = 0,$$

$$\lambda \frac{dT_1}{dx_1}\bigg|_{x_1=b-0} - \lambda \frac{dT_1}{dx_1}\bigg|_{x_1=b+0} - \lambda \frac{dT_3}{dx_3}\bigg|_{x_3=0} = 0. \quad (3)$$

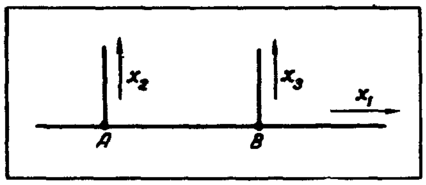

Fig. 2. Heat calculation diagram.

Thus, it is necessary to solve the boundary value problem (1) and (2) for a set of three ordinary differential second-order equations connected by condition (3). This is a fairly difficult computational problem, but, since the height of the ribs is insignificant in comparison with the transverse dimensions of the cover, we can assume as an approximation that the rib temperature is constant and replace the ribs by equivalent heat sinks and sources with due regard to the total amount of heat released in the rib and the heat removed from the surface of the rib. The problem then reduces to finding the solution of one ordinary nonlinear differential equation of the second order with specified boundary conditions,

$$S(x)\lambda(T)\frac{d^2T}{dx^2} + S(x)\frac{d\lambda}{dT}\left(\frac{dT}{dx}\right)^2 + \frac{dS}{dx}\lambda(T)\frac{dT}{dx} -$$
$$- k(T)S_1(x)(T-T_0) + \frac{I^2(x)\rho(T)}{S(x)}\eta = 0, \quad (4)$$

$$-\lambda\frac{dT}{dx}S\bigg|_{x=0} = q, \quad -\lambda\frac{dT}{dx}\bigg|_{x=x_k} - k(T-T_0)S\bigg|_{x=x_k} = 0. \quad (5)$$

The first boundary condition means that at the edge of the cover where current is being conducted there is a heat source due to contact resistance. The second boundary condition indicates that at the opposite edge of the cover heat exchange with the surroundings takes place. Function $\rho(T)$ has the usual form

$$\rho(T) = \rho_{20}[1 + \alpha(T - 20)].$$

Functions λ and k have the form [1]

$$\lambda(T) = 3.25(635 - T)\cdot 10^{-2} + 28.8,$$

$$k(T) = 2.8\sqrt[4]{T - T_0} + \frac{3.97[(T/100)^4 - (T/100)^4]}{T - T_0}.$$

The problem was solved on a Minsk-2 computer by the Runge-Kutta method. Since this method only solves systems of differential equations with initial conditions (the Cauchy problem), the program was compiled in the following way. The machine chose the temperature value on the left end so that when the first boundary condition was fulfilled the solution of the equation satisfied the boundary condition on the right end with a specified degree of accuracy. Hence we obtained the solution of the boundary problem (4), (5) with specified accuracy.

We note that problem (1) with conditions (2) and (3) can be solved in a similar manner by the Runge-Kutta method. This entails choosing the temperature value on the left end $T_1|_{x_1=0}$ so that conditions (2) and (3) are satisfied. In this case the algorithm of the solution is as follows.

A value $T_1|_{x_1=0}$ is assigned and a value of $(dT_1/dx_1)|_{x_1=0}$ satisfying the boundary condition

$$\left[\lambda\frac{dT_1}{dx_1} + f(T_1 x_1)\right]\bigg|_{x_1=0} = 0,$$

is determined. The first equation of system (1) for the region from $x_1 = 0$ to $x_1 = a$ (i.e. to the first branch point) is solved. The second equation of system (1) for the region from $x_2 = 0$ to $x_2 = l_2$ with initial conditions

$$T_2|_{x_2=0} = T_1|_{x_1=a},$$
$$\frac{dT_2}{dx_2}\bigg|_{x_2=0} = \alpha_1\frac{dT_1}{dx_1}\bigg|_{x_1=a}, \quad (5a)$$

is solved.

The method of successive approximations is used to choose a value of α_1 so that at $x_2 = l_2$ the boundary condition

$$\left[\lambda_2\frac{dT_2}{dx_2} + f_2(T_2, x_2)\right]\bigg|_{x_2=l_2} = 0,$$

is satisfied. We note that the upper limit of α_1, is determined from the condition that the maximum heat flux in the lower section of the rib cannot exceed the amount of heat removed by the rib from the cover with the rib temperature constant and equal to the temperature of its base,

$$T_2(x_2) = T_1(x_1)|_{x_1=a}.$$

Then from the condition that the total heat flux at the branch point is zero, we obtain

$$\frac{dT}{dx_1}\bigg|_{x_1=a+0} = (1-\alpha_1)\frac{dT_1}{dx_1}\bigg|_{x_1=a-0}. \quad (5b)$$

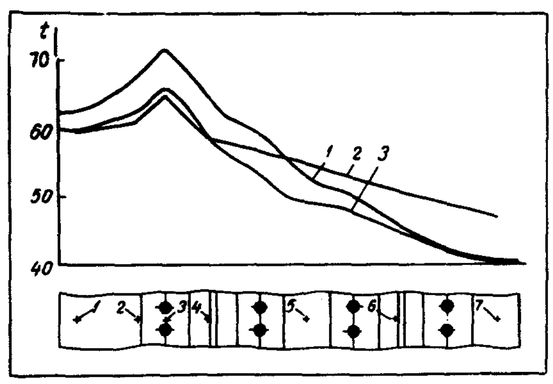

Fig. 3. Surface temperature distribution along cross section of cover: 1) Calculated without allowance for removal of heat by ribs; 2) experimental (from mean values); 3) calculated with allowance for removal of heat by ribs.

The first equation of the system for the region from $x_1 = a$ to $x_1 = b$ (i.e. to the second branch point) is solved with initial conditions

$$T_1(x_1)|_{x=a+0} = T_1(x_1)|_{x_1=a-0},$$

and (5b). The procedure carried out at the first branch point is repeated at the second branch point, i.e., the coefficient α_2 is determined. After this, the solution of the first equation of system (1) for the region from $x_1 = b$ to $x_1 = l_1$ with initial conditions

$$T_1(x_1)|_{x_1=b+0} = T_1(x_1)|_{x_1=b-0},$$

$$\left.\frac{dT_1}{dx_1}\right|_{x_1=b+0} = (1-\alpha_2)\left.\frac{dT_1}{dx_1}\right|_{x_1=b-0},$$

is completed. The calculation gives some values of $T_1(l_1)$ and $(dT_1/dx_1)(l_1)$ which may not satisfy the corresponding boundary condition on the right end of the cover. The whole solution procedure is then repeated with a new initial value of $T_1(x_1)|_{x_1=0}$. The method of successive approximations is used to find the value $T_1|_{x_1=0}$ at which the boundary condition on the right end ($x_1 = l_1$) is satisfied with specified accuracy.

We calculated (4) and (5) for several different current loads on the electrolytic bath (I = 75, 80, and 100 kA) and with different simplifying assumptions: a) the ribs are absent (usual one-dimensional nonlinear problem); b) the ribs are replaced by equivalent sinks and sources on the assumption that the temperature is constant over the rib height and is equal to the temperature at the base of the rib.

It is obvious that the solution of (4) and (5) with assumptions a and b gives upper and lower estimates. respectively, of the temperature on the surface of the cover $T_1(x_1)$, obtained by the solution of system (1)–(3). In fact, in the case of assumption a the removal of some of the heat from the rib is neglected and, hence, the cover temperature is overestimated. In the case of assumption b the heat transfer from the rib surface is overestimated and, hence, the temperature on the surface of the cover is less than the true value.

The results of the calculation are shown in Fig. 3. Curve 1 corresponds to assumption a and curve 3 to assumption b. As the figure shows, the greatest difference between the curves is about 5° C. Hence, in this case there is no need to seek an exact solution of problem (1)–(3), which must lie between curves 1 and 3.

Below we give the results of the experimental measurements and compare them with the results of calculation. The surface temperature was measured by an instrument of accuracy class 2.5 with a scale from 20 to 120° C.

The choice of the points for measurement of the temperature over the cross section of the cover was determined by its specific design: point 1 was located directly at the point of connection of the anode busbar; 2 lay in front of the first row of anode rods, at the edge of the rubber coating; 3 lay in the gap between the anode rods of the first row (the rubber coating at the measurement points was removed); 4 lay in front of the first rib, from which the current was fed through a wire to the first and second rows of anode rods; 5 lay in front of the third row of anode rods, at the edge of the rubber coating; 6 lay midway between the third row of anode rods; 7 lay after the fourth row of anode rods, at the edge of the rubber coating (Fig. 3).

A comparison of the results of calculation and measurements showed that the calculated curves were in good agreement with the experimental data. A characteristic feature was the increase in temperature in a very narrow section of the cover (in the gap between the anode rods of the first row). The difference of several degrees C in the hot part of the cover must be attributed to the inaccuracy of the chosen coefficients and to the simplifying assumptions regarding the ribs. The higher value of the actual temperature of the cover in comparison with the calculated value in the less hot region can be attributed to the assumed absence of heat transfer between the underside of the cover and the gas inside the electrolytic bath (the temperature difference here is 30–40° C). The agreement between the calculated and experimental temperatures is sufficiently good to make the proposed method of calculation suitable for the design of electrolytic baths.

The temperature distribution over the cover should be calculated as a check after its main dimensions have been determined from considerations of strength and a suitable shape has been adopted. At this stage the initial form of functions S(x) and $S_1(x)$ is assigned. If the heat calculation shows the presence of regions with temperature above the permissible value, the design or the current switching system must be altered and the calculation performed again. These operations are continued until a satisfactory result is obtained. To allow for the action of equalizing currents passing along the axis of the cover we recommend assigning an increased current load to one element of the equivalent circuit.

NOTATION

$S(x_i)$ is the cross-sectional area of the corresponding region of rod; ρ is the resistivity of the cover material; λ is the thermal conductivity; I_i is the current in the corresponding region of the cover; T_i is the temperature, °C; η is the conversion coefficient, equal to 0.86 J/kcal; k is the heat transfer coefficient; S_1 is the area of heat-losing surface; T_0 is the ambient temperature, °C.

REFERENCE

1. S. S. Kutateladze and V. M. Borishanskii, Heat Transfer Manual [in Russian], Gosenergoizdat, 1959.

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